Determination of c and b quark masses

Christine Davies
SUPA, School of Physics and Astronomy
University of Glasgow
Glasgow G12 8QQ, U.K.

1 Introduction

The determination of quark masses has been transformed in the past few years by accurate results from realistic lattice QCD. This has meant a range of new methods for both b and c quarks [1] which I will describe along with results from continuum techniques. The recent improvement in Lattice QCD actions along with the generation of gluon configurations including sea quarks on fine lattices has given us viable methods for handling heavy quarks with relativistic actions. This has allowed us for the first time to connect the heavy and light sectors through accurate determination of ratios of quark masses, such as m_c/m_s [2]. Such ratios enable us to leverage the accuracy of c and b mass determinations into accurate s and light quark masses. The comparison of quark masses from different methods and formalisms provides a strong test of QCD and the masses themselves are needed in calculations for the cross-sections of various processes at LHC, for example $H \to b\bar{b}$.

2 Lattice QCD methods

Lattice QCD calculations have the advantage over the real world of having direct access to the parameters of the QCD Lagrangian. These are tuned against experimentally well-determined hadron masses, one for each quark mass. The masses in the lattice QCD Lagrangian are in a lattice regularisation scheme (which depends on the discretisation of the QCD Lagrangian used) and at a scale which depends on the inverse lattice spacing. To convert them to the standard $\overline{MS}$ scheme at a fixed scale must be done either directly, by determining a renormalisation constant for the conversion, or indirectly, by calculating on the lattice some ultraviolet-finite quantity whose value is known in the continuum in terms of the $\overline{MS}$ quark mass.

The direct conversion, $\overline{m}(\mu) = Z(\mu a)m_{latt}(1/a)$, either calculates Z in lattice QCD perturbation theory or matches lattice calculations to a MOM-scheme definition that can then be converted to $\overline{MS}$ using continuum QCD perturbation theory.

A well-developed and accurate indirect method for heavy quarks is the current-current correlator method [3]. Here the continuum limit of time-moments of a well-defined lattice QCD heavyonium correlator is compared to continuum QCD perturbation theory for Q^2-derivative moments of the heavy quark vacuum polarization function. The perturbation theory is known through α_s^3 for some moments since this method has been developed for accurate calculations in the continuum - see, for example [4]. There m_b and m_c are extracted from experimental data by isolating the b and c quark contributions to $\sigma(e^+e^- \to \text{hadrons})$ [4].

Handling b and c quarks in lattice QCD is made difficult by the presence of potentially large discretisation errors when the heavy quark mass in lattice units, $m_Q a$, is $\mathcal{O}(1)$. These errors can be beaten down by systematically improving the discretisation of the Dirac Lagrangian and this is the best approach for c quarks. For example, the Highly Improved Staggered Quark (HISQ) action [5] gives only few % errors for c physics at lattice spacings, $a \approx 0.1\,\text{fm}$. Using this same approach for b physics [6] requires very fine lattices and extrapolation to the b quark mass from smaller masses where discretisation errors are under better control, but has the advantage that the same lattice action can be used for all 5 quarks from u to b. Alternatively for b quarks there are a number of discretised effective theory approaches that avoid large discretisation errors because the b quarks are nonrelativistic. The price to be paid, however, is a more complicated action that needs more renormalisation.
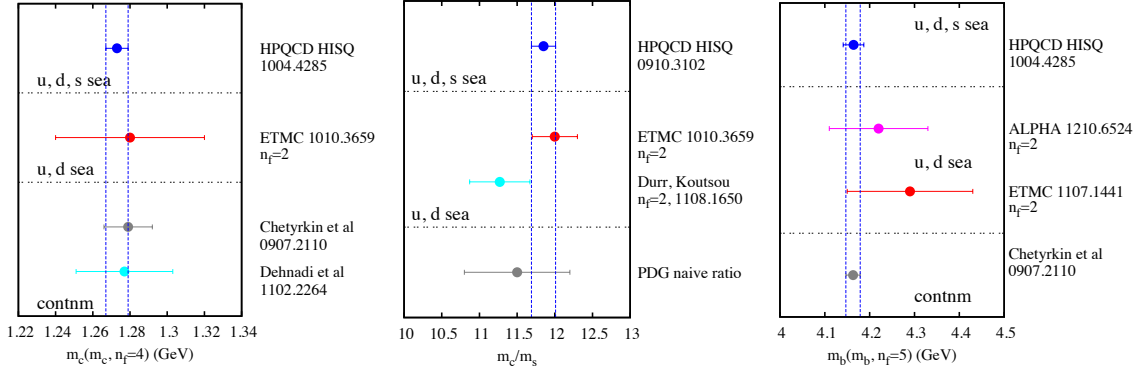

Figure 1: Comparison plots for (from left to right): m_c, m_c/m_s and m_b. Masses are in the $\overline{MS}$ scheme with a scale equal to the mass itself. The ratio is scale-invariant. The lattice QCD results marked as 'u, d sea' do not include s quarks in the sea and therefore may not be comparable to the other numbers.

3 Results

Figure 1 compares results from lattice QCD and continuum determinations for m_c (left) and m_b (right). For m_c 1% errors are possible and for m_b, 0.5%. The most accurate values are: $\overline{m}_c(\overline{m}_c) = 1.273(6)\,\mathrm{GeV}$ [6] and $\overline{m}_b(\overline{m}_b) = 4.163(16)\,\mathrm{GeV}$ [4].
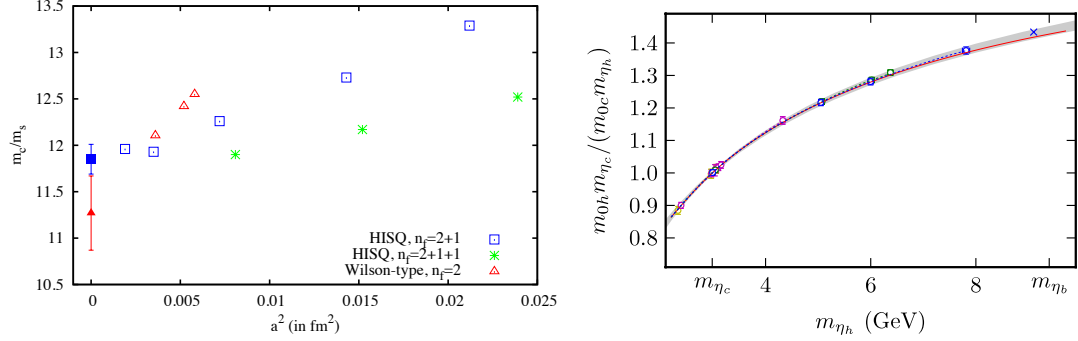

Figure 2: The figure on the left shows the ratio m_c/m_s plotted against the square of the lattice spacing for results using HISQ quarks on $n_f = 2+1$ configurations [2] (blue open squares) and Wilson-type quarks on $n_f = 2$ configurations [15] (red open triangles). Errors are only shown on the final extrapolated result (filled symbol) at $a = 0$. Green bursts show preliminary HISQ results on $n_f = 2+1+1$ configurations [16]. The figure on the right shows the ratio m_h/m_c, divided by the ratio of the pseudoscalar meson masses, plotted as a function of the pseudoscalar heavyonium mass in the range from $m_h = m_c$ to $m_h = m_b$ [6].

In the plot for m_c, the result from the ETM Collaboration [7] is a direct determination using the twisted mass quark formalism [8] on gluon configurations including only u/d quarks in the sea ($n_f = 2$) and a Z factor determined using the MOM scheme. Note that this result is $m_c(n_f = 2)$ and so may differ from the other results since there is no perturbative way of correcting for missing sea s quarks. The other results use the current-current correlator method. The HPQCD result [6] uses HISQ quarks on gluon configurations including sea u, d and s quarks ($n_f = 2+1$) comparing time-moments of the absolutely normalised (in this formalism) pseudoscalar charmonium correlators to continuum QCD perturbation theory. The $n_f = 3$ quark mass is converted to $n_f = 4$ perturbatively. The lower two results [4, 9] are from comparing continuum QCD perturbation theory to moments of the c quark contribution (effectively via the vector charmonium correlator) to $\sigma(e^+e^- \to \mathrm{hadrons})$. The two results differ in their treatment of errors, particularly the perturbative error which is handled in both cases by varying the scale at which α_s is determined. Both have a significant error from the value of α_s. The lattice QCD result from HPQCD [6] has an advantage here in that α_s is simultaneously determined along with m_c (and m_b).

3

A particularly accurate extraction of α_s is possible from the lowest moment of the pseudoscalar correlator, which has no explicit mass dependence. In fact the lattice calculation fits the 4 lowest moments simultaneously and for a range of masses from c to b. This allows both a constraint on higher order terms in perturbation theory beyond α_s^3 and a determination of the perturbative error from the effect of including them. Moments of the lattice QCD vector correlator can be compared directly to the results extracted from $\sigma(e^+e^- \to \text{hadrons})$ [4] and this provides a 1.5% test of QCD [10] (see also [11]).

The rightmost plot of Figure 1 compares results for m_b. The top value from HPQCD/HISQ [6] uses the current-current correlator method and is determined in the same analysis as m_c above. The bottom result is also from the same continuum determination as the m_c above [4], using e^+e^- data in a different region of $\sqrt{s}$. These are the most accurate results. The two central lattice QCD results use different techniques and so far both include only u/d quarks in the sea, so the results may not be comparable. The Alpha collaboration [12] uses static (infinite mass) quarks but with $1/m_Q$ corrections and determines the b mass fom the binding energy of a heavy-light meson using nonperturbative step-scaling to determine the energy offset. The ETM collaboration [13] interpolates the ratio of the heavy-light meson mass to the quark mass between relativistic twisted mass quarks and static quarks. A direct determination of m_b is available from lattice NRQCD on gluon configurations that include u, d, s and c quarks in the sea, but it uses a Z factor only determined to $\mathcal{O}(\alpha_s)$ in lattice QCD perturbation theory so is rather inaccurate [14].

Figure 2 shows the quark mass ratios m_c/m_s and m_b/m_c determined from lattice QCD calculations. The ratio of lattice QCD masses translates directly to the ratio of $\overline{MS}$ masses (at the *same* scale) because the Z factors cancel when the same formalism is used for both quarks. This is then a fully nonperturbative determination and can be done with an accuracy of 1%.

The leftmost plot shows results for m_c/m_s as a function of a^2 for HISQ [2] and Wilson-type quarks [15]. The improved a-dependence of HISQ quarks is evident, particularly for new results on $n_f = 2 + 1 + 1$ configurations that include a further improved gluon action and HISQ quarks in the sea [16]. A comparison of different lattice QCD results for m_c/m_s is given in the middle plot of Figure 1, compared to the ratio obtained from the Particle Data Tables [17] running m_c and m_s to the same scale. The best, and only $n_f = 2 + 1$, result is $m_c/m_s = 11.85(16)$ from HPQCD [2]. It can be used with an accurate value for m_c to obtain a 1% accurate m_s [6].

The rightmost plot of Figure 2 shows HISQ results for m_h/m_c over the range of heavy quark masses from c to b. Very little a-dependence is seen and a 1% accurate result at the b can be obtained: $m_b/m_c = 4.49(4)$ [6]. This agrees well with the ratio of masses obtained from the current-current correlator method and provides a good nonperturbative test of that method.

4

References

[1] C. Davies, PoS LATTICE **2011**, 019 (2011) [arXiv:1203.3862 [hep-lat]].

[2] C. T. H. Davies *et al* [HPQCD Collaboration], Phys. Rev. Lett. **104**, 132003 (2010) [arXiv:0910.3102 [hep-ph]].

[3] I. Allison *et al.*, Phys. Rev. D **78**, 054513 (2008) [arXiv:0805.2999 [hep-lat]].

[4] K. G. Chetyrkin, J. H. Kuhn, A. Maier, P. Maierhofer, P. Marquard, M. Steinhauser and C. Sturm, Phys. Rev. D **80**, 074010 (2009) [arXiv:0907.2110 [hep-ph]].

[5] E. Follana *et al.* [HPQCD Collaboration], Phys. Rev. D **75**, 054502 (2007) [hep-lat/0610092].

[6] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel, G. P. Lepage [HPQCD Collabration], Phys. Rev. D **82**, 034512 (2010) [arXiv:1004.4285 [hep-lat]].

[7] B. Blossier *et al.* [ETM Collaboration], Phys. Rev. D **82**, 114513 (2010) [arXiv:1010.3659 [hep-lat]].

[8] R. Frezzotti *et al.* [Alpha Collaboration], JHEP **0108**, 058 (2001) [hep-lat/0101001].

[9] B. Dehnadi, A. H. Hoang, V. Mateu, S. M. Zebarjad, arXiv:1102.2264 [hep-ph].

[10] G. C. Donald *et al* [HPQCD Collaboration], Phys. Rev. D **86**, 094501 (2012) [arXiv:1208.2855 [hep-lat]].

[11] K. Jansen, M. Petschlies and C. Urbach, PoS LATTICE **2011**, 234 (2011) [arXiv:1111.5252 [hep-lat]].

[12] F. Bernardoni *et al.* [ALPHA Collaboration], arXiv:1210.6524 [hep-lat].

[13] P. Dimopoulos *et al.* [ETM Collaboration], JHEP **1201**, 046 (2012) [arXiv:1107.1441 [hep-lat]].

[14] R. J. Dowdall *et al.* [HPQCD Collaboration], Phys. Rev. D **85**, 054509 (2012) [arXiv:1110.6887 [hep-lat]].

[15] S. Durr and G. Koutsou, Phys. Rev. Lett. **108**, 122003 (2012) [arXiv:1108.1650 [hep-lat]].

[16] R. J. Dowdall, C. T. H. Davies, T. C. Hammant and R. R. Horgan [HPQCD Collaboration], Phys. Rev. D **86**, 094510 (2012) [arXiv:1207.5149 [hep-lat]].

[17] J. Beringer *et al* [Particle Data Group], Phys. Rev. D**86**:010001 (2012).